Exploration of long records of extreme rainfall and design rainfall inferences

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1. Introduction

According to the statistical theory of extremes, the distribution function $H(x)$ of the maximum of a number n of identically distributed random variables, if n is large enough (theoretically infinite), takes the asymptotic form, known as the Generalised Extreme Value (GEV) distribution (Jenkinson, 1955),

$$H(x) = \exp\left\{-\left[1+\kappa\left(\frac{x}{\lambda}-\psi\right)\right]^{-1/\kappa}\right\}, \qquad \kappa\, x \geq \kappa\, \lambda\, (\psi - 1/\kappa) \tag{1}$$

where ψ, $\lambda > 0$ and κ are location, scale and shape parameters, respectively. (Note that the sign convention of κ in (1) is opposite to that most commonly used in hydrological texts and the location parameter is dimensionless).

When $\kappa = 0$, the type I distribution of maxima (EV1 or Gumbel distribution),

$$H(x) = \exp[-\exp(-x/\lambda + \psi)] \tag{2}$$

is obtained, which is unbounded from both below and above ($-\infty < x < +\infty$). When $\kappa > 0$, $H(x)$ represents the extreme value distribution of maxima of type II (EV2), which is bounded from below and unbounded from above ($\lambda\,\psi - \lambda/\kappa \leq x < +\infty$). A special case, the Fréchet distribution, is obtained when the lower bound becomes zero ($\psi = 1/\kappa$). When $\kappa < 0$, $H(x)$ represents the type III (EV3) distribution of maxima. This, however, is of no practical interest in hydrology as it refers to random variables bounded from above ($-\infty < x \leq \lambda\,\psi - \lambda/\kappa$). It is noted that if the distribution of minima is of interest, the roles of types II and III reverse, e.g. the type III distribution is bounded from below and is significant for the study of droughts.

The EV1 distribution has been the prevailing model for rainfall extremes despite the fact that it results in the highest possible risk for engineering structures, i.e. it yields the smallest possible design rainfall values in comparison to those of the EV2 for any value of the shape parameter. The simplicity of the calculations of the EV1 distribution along with its geometrical depiction through a linear probability plot may have contributed to its popularity in hydrologists and engineers. There is also a theoretical justification, as EV1 is the asymptotic extreme value distribution for a wide range of parent distributions that are common in hydrology.

However, a recent study (Koutsoyiannis, 2003) showed that convergence of the exact distribution of maxima to the asymptote EV1 may be extremely slow, thus making the EV1 distribution an inappropriate approximation of the exact distribution of maxima. Besides, the attraction of parent distributions to this asymptote relies on the assumption that parameters of the parent distribution are constant in time, which may not be the case in hydrological processes. Slight relaxation of this assumption may result in the EV2 rather than the EV1 asymptote. It was also shown that small sizes of records, typically used in hydrological applications, hide the EV2 distribution and display it as if it were EV1. This allowed the conjecture that the broad use of the EV1 distribution worldwide may in fact be related to small sample sizes rather than to the real behaviour of rainfall maxima, which should be better described by the EV2 distribution. This conjecture is investigated thoroughly here based on real-world long rainfall records.

2. Data

Some thousands of raingauge data sets from Europe and USA were examined, namely data from the United States Historical Climatology Network (USHCN), Land Surface Observation Data of the UK Met Office, and data from the oldest stations of France, Italy and Greece. Among these, a total of 169 stations were found to have at least 100 years of data (not including the years with missing data) and were chosen for further analysis. From the continuous record of each station, the series of annual maximum values of daily rainfall were extracted. The geographical locations of stations are shown in Figure 1 and are classified in six geographical zones as shown in Table 1. Table 2 gives the general characteristics of the top ten, in terms of record length, raingauges, which are from all countries of this case study (USA, UK, France, Italy and Greece).

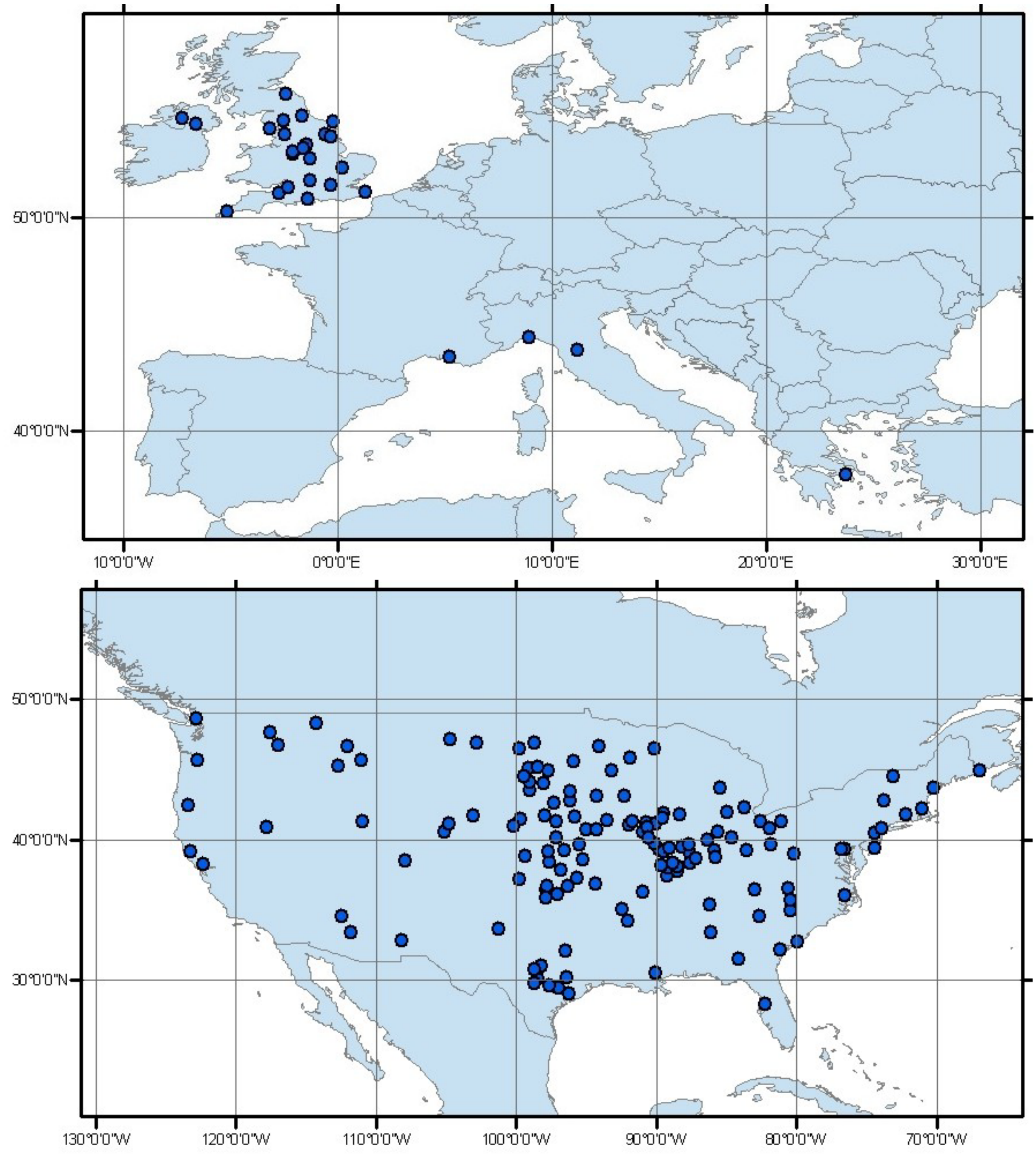

Figure 1 Geographical locations of raingauges.

3. Initial exploration

As an initial step, the typical statistics and the L-statistics (based on probability-weighted moments; Hosking 1990) were estimated for each annual maximum series of daily rainfall depths. The ranges in each of the six geographical zones and the corresponding averages over all stations of each zone are shown in Table 3. Potential relationships among these statistics and their geographical variation were subsequently explored.

Table 1 Geographical zones and corresponding numbers of raingauges and data values.

Zone no.	Description	Number of stations	Number of station-years
1	USA, N of the 35th parallel and E of the 105th meridian	104	10942
2	USA, S of the 35th parallel and E of the 105th meridian	19	2012
3	USA, N of the 35th parallel and W of the 105th meridian	15	1610
4	USA, S of the 35th parallel and W of the 105th meridian	3	304
5	UK	24	2624
6	Mediterranean: Greece, Italy, S France	4	573
Total		169	18065

Table 2 General characteristics of the top ten, in terms of record length, raingauges.

Name	Zone	Latitude (°N)	Longitude (°)	Elevation (m)	Record length	Start year	End year	Years with missing values
Florence	6	43.80	11.20	40	154	1822	1979	1874-77
Genoa	6	44.40	8.90	21	148	1833	1980	
Athens	6	37.97	23.78	107	143	1860	2002	
Charleston City	2	32.79	–79.94	3	131	1871	2001	
Oxford	5	51.72	–1.29		130	1853	1993	1930, 1933,1961-69
Cheyenne	1	41.16	–104.82	1867	130	1871	2001	1877
Marseille	6	43.45	5.20	6	128	1864	1991	
Armagh	5	54.35	–6.65		128	1866	1993	
Savannah	2	32.14	–81.20	14	128	1871	2001	1969-71
Albany	1	42.76	–73.80	84	128	1874	2001	

Table 3 Statistical characteristics of annual maximum daily rainfall series for the different geographical zones.

		Geographical zone						
		1	2	3	4	5	6	Total
Sample mean, μ (mm)	min	34.2	65.3	19.1	31.8	31.3	48.5	19.1
	mean	65.7	91.0	36.5	39.4	36.1	68.9	61.4
	max	90.1	109.0	75.3	48.7	46.4	110.9	110.9
Sample maximum, $x_{\max}$ (mm)	min	88.4	146.8	40.1	84.3	54.1	140.0	40.1
	mean	175.8	265.7	83.9	125.2	89.7	225.4	165.8
	max	429.5	490.0	157.0	201.2	130.3	389.2	490.0
Coefficient of variation, C_v	min	0.26	0.32	0.31	0.35	0.26	0.35	0.26
	mean	0.38	0.42	0.36	0.41	0.34	0.42	0.38
	max	0.68	0.57	0.47	0.47	0.45	0.48	0.68
Coefficient of skewness, C_s	min	0.58	0.89	0.83	1.08	0.55	1.65	0.55
	mean	1.69	1.81	1.19	1.93	1.70	1.92	1.67
	max	4.94	3.89	1.69	3.32	3.22	2.03	4.94
L-coefficient of variation, τ_2	min	0.14	0.18	0.16	0.19	0.14	0.18	0.14
	mean	0.19	0.21	0.19	0.21	0.17	0.22	0.19
	max	0.26	0.25	0.25	0.22	0.22	0.24	0.26
L-coefficient of skewness, τ_3	min	0.12	0.16	0.14	0.16	0.15	0.22	0.12
	mean	0.24	0.26	0.21	0.23	0.24	0.26	0.24
	max	0.43	0.38	0.26	0.29	0.35	0.28	0.43

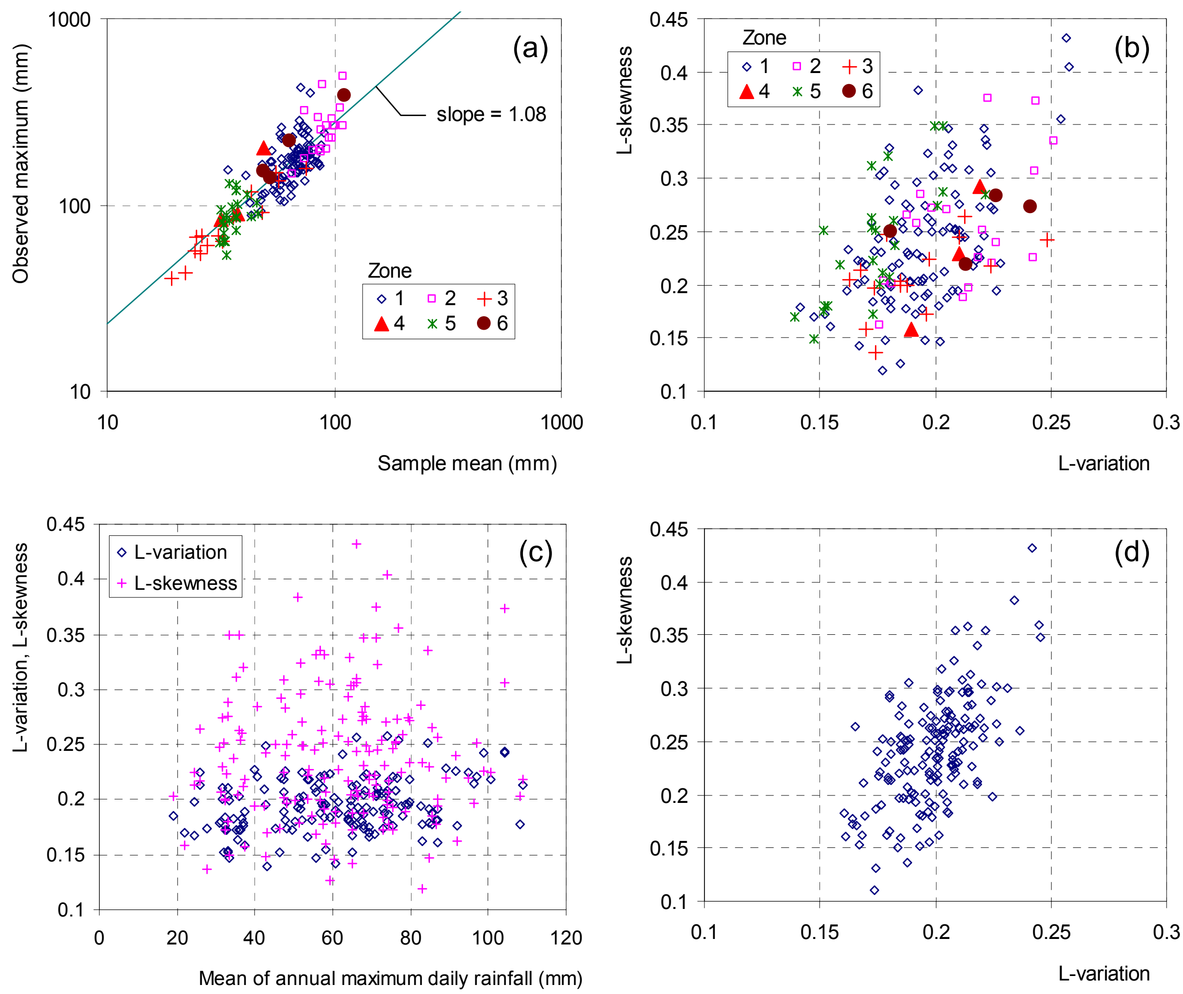

Figure 2 (a) Sample mean and maximum, over the observation period, of each annual maximum daily rainfall series for the six different geographical zones; (b) L-variation and L-skewness coefficients vs. the mean of the annual maximum series; (c) L-skewness vs. L-variation coefficient for the series of the six different geographical zones; (d) L-skewness vs. L-variation coefficient for 169 synthetic samples with lengths and means equal to those of the historical records, generated from the EV2 distribution with shape parameter $\kappa = 0.103$ and location parameter $\psi = 3.34$.

The sample mean (μ) and maximum values (x_{max}) over the observation period of each annual maximum daily rainfall series are plotted in Figure 2(a) using different symbols for each geographical zone. It is observed there that (a) both mean and maximum values vary with the geographical zone (as expected): clouds of points referring to different zones occupy different areas in the diagram; (b) there exists a clear relationship between mean and maximum values (as expected), which seems to be independent of the geographical zone; and (c) this relationship can be approximated by a power law with exponent slightly higher than one. The coefficients of L-variation (τ_2) and L-skewness (τ_3), plotted in Figure 2(b) against mean, are highly variable and do not correlate with mean. Figure 2(c), in which τ_3 is plotted against τ_2 using different symbols for different geographical zones, shows a positive correlation between τ_2 and τ_3 (correlation coefficient = 0.52) and simultaneously indicates independence of the geographical zone, as clouds of points referring to different zones are homogeneously mixed in the diagram. However, the positive correlation between τ_2 and τ_3 does not have a physical meaning but rather is a statistical effect. To show this, 169 synthetic samples with lengths and means equal to those of the historical records, were generated from the EV2 distribution with constant shape parameter $\kappa = 0.103$ and location parameter $\psi = 3.34$ (these values are clarified in the next section). The values of the statistics τ_2 and τ_3 of these synthetic samples have been plotted in Figure 2(d), which reveals a picture similar to that of Figure 2(c) with a strong correlation between τ_2 and τ_3 (correlation coefficient = 0.60). Notably, the dispersion of τ_3 in Figure 2(d) is identical to that in Figure 2(c) whereas the dispersion of τ_2 in the former is slightly smaller than in the latter.

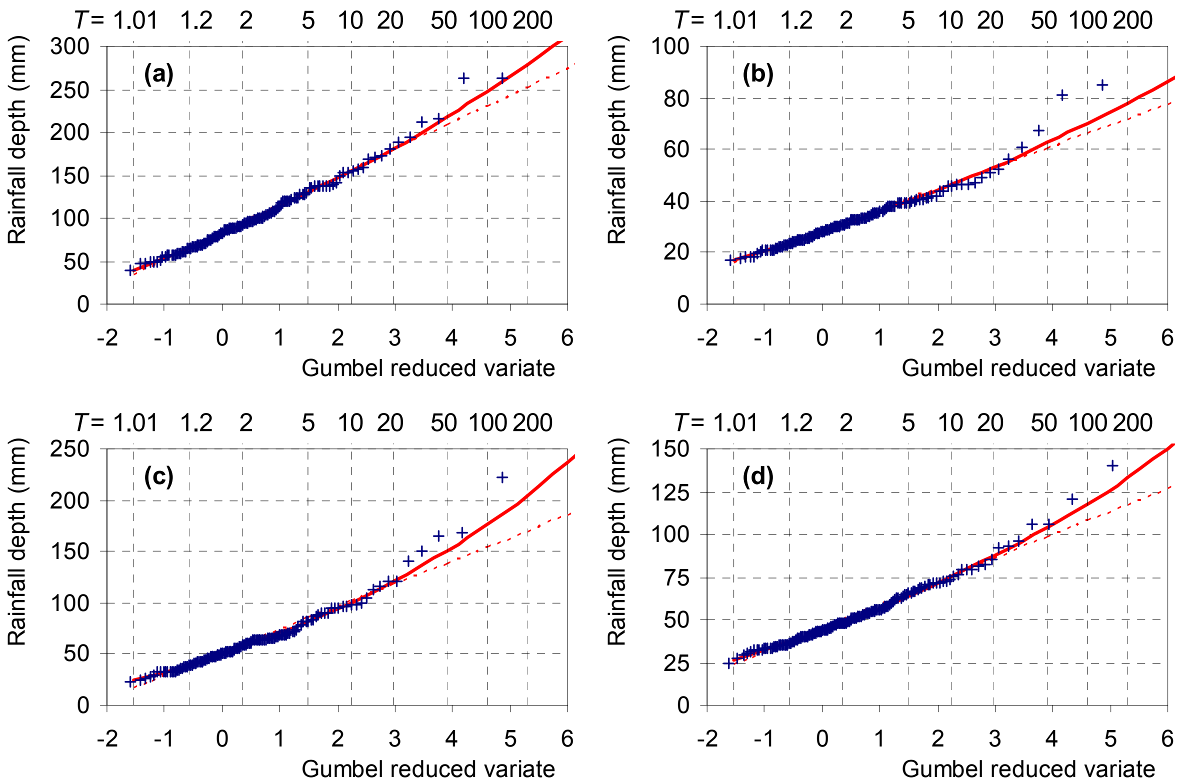

Figure 3 EV2 (continuous lines) and EV1 (dotted lines) distributions fitted by the method of L-moments and comparison with the empirical distribution (crosses) for the annual maximum series of (a) Charleston City, USA/SC; (b) Oxford, UK (c) Marseille, France; and (d) Florence, Ximeniano Observatory, Italy (Gumbel probability plots). The estimated shape parameters κ are respectively 0.083, 0.081, 0.155 and 0.120.

4. Fitting of distribution functions

GEV distributions were fitted to each of the 169 series using three methods, maximum likelihood, moments and L-moments. The averages over all raingauges and the dispersion characteristics (minimum and maximum values and standard deviations) of the parameters are shown in Table 4. The shape parameter κ is the most important as it determines the type of the distribution of maxima (EV1 or EV2) and consequently the behaviour of the distribution in its tail. Besides, it is the most uncertain parameter as its estimation depends on the skewness whose value cannot be determined accurately. Clearly, Table 4 shows that in more than 90% of the series the estimated κ is positive, which suggests EV2 distributions. (The smaller values of κ given by the method of moments, as shown in Table 4, and the smaller percentage of positive values, 74%, are clearly a result of the significant negative bias implied by the estimators of this method, as verified by Monte Carlo simulations). The estimated κ values range between slightly negative to over 0.30. Given the observation of the previous section on Figure 2(d) regarding the statistical behaviour of τ_3, which determines κ, it should be expected that the large range of κ values is rather a statistical effect. This will be examined further below.

Figure 3 depicts the EV2 distribution functions fitted by the method of L-moments to four of the stations included in the top ten stations of Table 2. For comparison, Figure 3 includes also plots of the EV1 distributions fitted again by the method of L-moments and of the empirical distributions determined using Weibull plotting positions. Clearly, the observed maxima for high return periods are higher than the predictions of the EV1 distributions and even from those of the EV2 distribution. Obviously, however the EV2 distribution is in closer agreement to the empirical distribution than the EV2 distribution. The differences of EV1 and EV2 seem to be not very significant for return periods up to 50-100 years.

These differences, however, become extremely significant when the distribution functions are extrapolated to higher return periods, which are greatly important in the design of major hydraulic constructions such as dam spillways. This is demonstrated in Figure 4(a), which is similar to the diagrams of Figure 3 but with emphasis given to the tail of the distribution, for return periods higher than 200 years. Figure 4(a) refers to another station, Athens, Greece, again included in the top ten stations of Table 2. Clearly, the EV1 distribution underestimates seriously the maximum rainfall for high return periods. For instance, at the return period 20 000 years the EV1 distribution results in a value of rainfall depth half that obtained by the EV2 distribution. Another comparison of the two distributions can be done in terms of the value of probable maximum precipitation (PMP). This was initially considered to be the greatest depth of precipitation for a given duration that is physically possible over a geographical location. However, more recently it has been considered as one high rainfall value that has a certain return period like other, higher or lower, values of rainfall depth. Thus, National Research Council (1994, p. 14) assessed that PMP estimates in the USA have return periods of the order 10^5 to 10^9 and Koutsoyiannis (1999) showed that PMP values estimated by the method of Hershfield (1961) have return periods around 60 000 years. The latter method was used by Koutsoyiannis and Baloutsos (2000) to estimate PMP in Athens and resulted in a value of 424.1 mm, which has been plotted in Figure 4(a). If the return period of this value is estimated by the EV2 distribution, it turns out to be 37 000 to 300 000 years depending on the parameter estimation method whereas EV1 results in the unrealistically high value 4×10^{10} years.

Table 4 Averages over all raingauges and dispersion characteristics of the parameters of the GEV distribution of the annual maximum daily rainfall series.

		Estimation method		
Parameter		Max likelihood	Moments	L-Moments
κ	Mean	0.103	0.052	0.103
	Standard deviation	0.080	0.079	0.085
	Min	–0.061	–0.121	–0.080
	Max	0.303	0.238	0.373
	Percent positive	91%	74%	92%
λ	Mean	15.39	16.64	15.52
	Standard deviation	5.63	6.31	5.81
	Min	4.95	5.16	4.86
	Max	31.08	34.89	32.13
ψ	Mean	3.36	3.14	3.34
	Standard deviation	0.42	0.44	0.43
	Min	2.54	2.07	2.42
	Max	4.48	4.44	4.47

5. Study of the variation of parameters

The problem of the parameter variation and the question whether this variation corresponds to physical (climatological) reasons or is a purely statistical (sampling) effect have been already posed in the previous sections. Here they will be studied more systematically. As already indicated, simulation assuming that one or more statistical parameters are constant is a proper means to assess the sampling effect and estimate the portion of parameter variation that this effect explains.

As already discussed, the variation of the means of the annual maximum daily rainfall series reflects a climatic variability and is different in different geographical zones. This is also verified by simulation: the standard deviation of means over all stations is 20.0 mm while a simulation assuming constant mean over all stations would yield a standard deviation of only 2.3 mm. This, however, is not the case with other parameters, if they are expressed on a non-dimensionalised basis. Their variability is mostly a sampling effect. To demonstrate this, 169 synthetic samples with lengths and means equal to those of historical series were generated from the GEV distribution with constant shape parameter κ = 0.103 and location parameter ψ = 3.34. These constant values are the averages of the relevant parameters estimated by the method of L-moments (Table 4). The empirical distributions of several dimensionless sample statistics were then obtained and compared graphically to the corresponding empirical distributions obtained from the 169 historical annual maximum daily rainfall series (Figure 5). It can be observed that in most cases the empirical distributions of the synthetic samples are almost identical to those of the historical ones. The highest differences between the two appear in the distributions of coefficients of variation τ_2 and C_v, and that of the location parameter ψ.

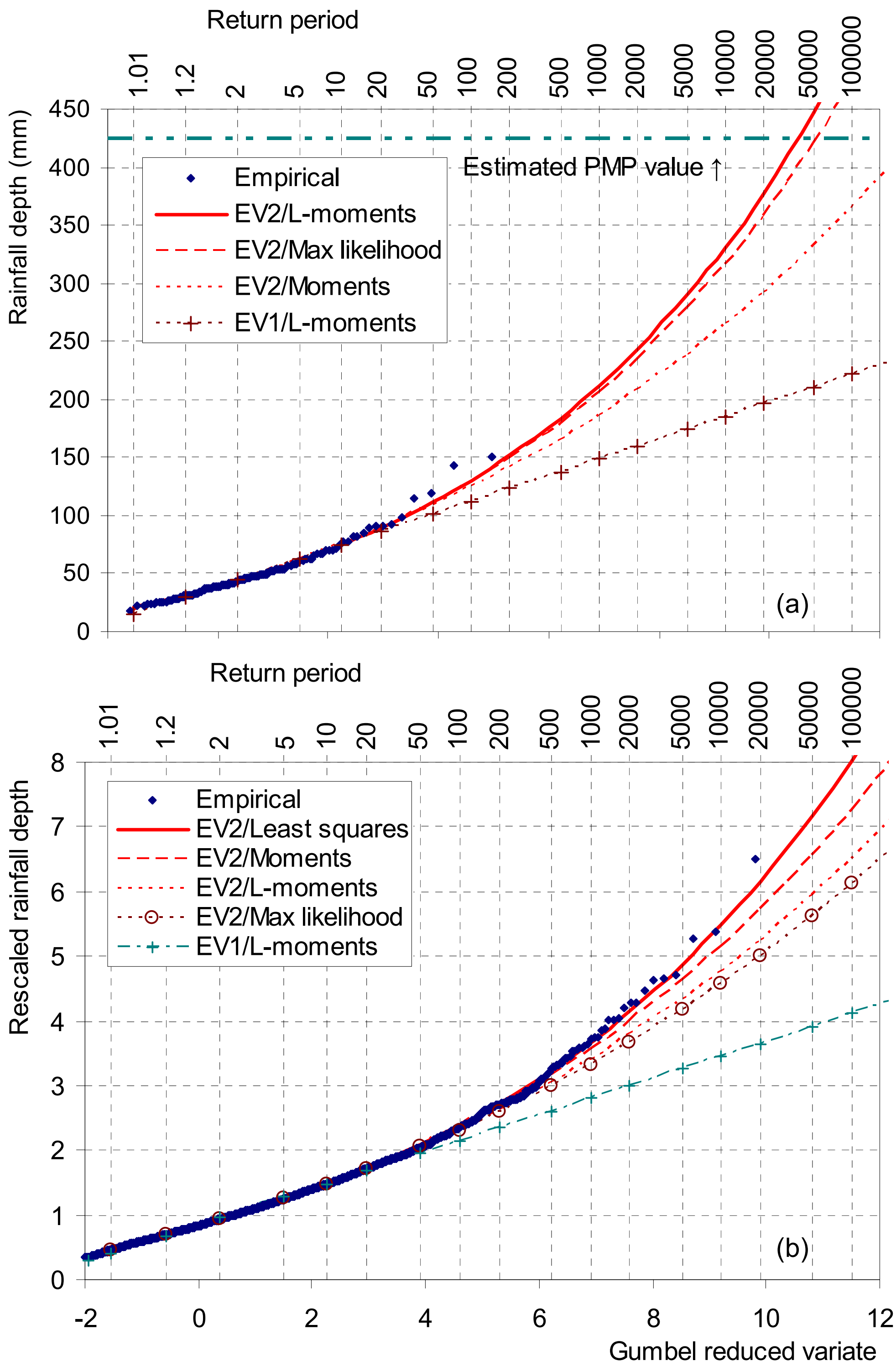

Figure 4 EV2 and EV1 distributions (Gumbel probability plots) fitted by several methods and comparison with the empirical distribution (a) for the series of Athens National Observatory, Greece (κ = 0.170, 0.158 and 0.106 for the methods of L-moments, maximum likelihood and moments respectively; PMP = 424.1, mm estimated by Koutsoyiannis and Baloutsos, 2000); (b) for the unified record of all 169 series.

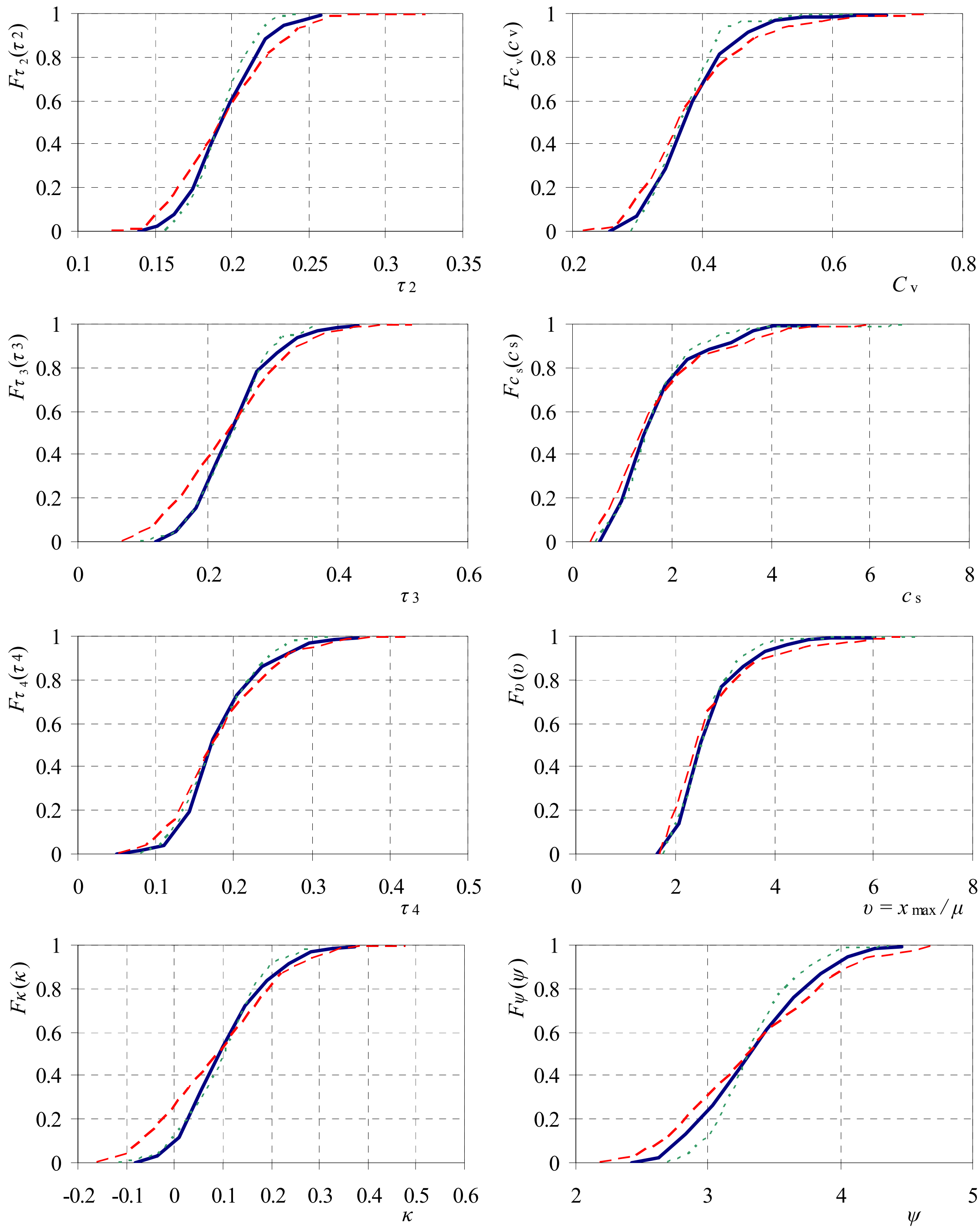

Figure 5 Empirical distribution functions of several dimensionless sample statistics (coefficients of variation τ_2 and C_v, skewness τ_3 and C_s, and kurtosis τ_4; ratio of maximum value x_{max} to mean value μ, denoted as υ; and L-moments estimates of parameters κ and ψ of the GEV distribution), as computed from either: the 169 historical annual maximum daily rainfall series (thick continuous lines); 169 synthetic samples with lengths and means equal to those of historical series generated from the GEV distribution with constant shape parameter $\kappa = 0.103$ and location parameter $\psi = 3.34$ (dotted lines); and 169 synthetic samples with lengths and means equal to those of historical series generated from the GEV distribution with shape parameter κ and location parameter ψ varying following uniform distributions (dashed lines).

An additional simulation was performed assuming that the parameters κ and ψ are not constant but random variables uniformly distributed over an interval, determined so as to match the standard deviation of the parameters shown in Table 4. The resulting empirical distribution functions are also plotted in Figure 5. In all cases, the greater dispersion of the simulated sampling distributions as compared to the historical ones is apparent.

Table 5 Parameters of the EV2 distribution as estimated by four different methods from the unified record of all 169 series.

	Estimation method			
Parameter	Max likelihood	Moments	L-moments	Least squares
κ	0.093	0.126	0.104	0.148
λ	0.258	0.248	0.255	0.236
ψ	3.24	3.36	3.28	3.54

These simulation experiments and comparisons with historical data suggest that a hypothesis of a common statistical law applying to all 169 series, except for a scaling parameter to account for the different means μ, is not far from reality. In this case, a radically improved approach to fitting a probability distribution becomes possible. If the annual maximum daily rainfall series of each station is rescaled by an appropriate scaling factor, then all 18 065 station-years can be regarded as realisations of the same statistical law and can be unified in one statistical record. The scaling factor can be the sample mean μ or a variant of it like the one used by Hershfield (1961) to take account of the effect of outliers on the sample mean.

The empirical distribution of the unified rescaled annual maximum daily rainfall series is depicted in Figure 4(b). To this, the EV2 distribution is fitted and also plotted in Figure 4(b) whereas its parameters are shown in Table 5. It is observed that the methods of maximum likelihood, moments and L-moments result in (a) different parameter estimates despite the extremely large record length (18 065), and (b) estimates of distribution quantiles that are systematically lower than the empirical estimates in the tail (for return periods > 500 years). Both these observations may indicate that the EV2 distribution is an imperfect model for extreme rainfall. However problem (b) can be resolved by adopting a different parameter estimation method. Here a weighted least squares method was used, which minimises the weighted average of square errors between empirical and EV2 quantiles. To give higher importance to the high values, weights equal to the empirical quantiles were assumed. As shown in Table 5, the latter method resulted in a shape parameter $\kappa = 0.15$, greater than those of the other methods. Its behaviour in the tail seems to be much closer to reality than those of the other methods (Figure 4(b)), which is very important from an engineer's point of view.

In addition to EV2, the EV1 distribution with parameters fitted by the method of L-moments ($\lambda = 0.283$, $\psi = 2.99$) was plotted in Figure 4(b). Its inappropriateness for return periods greater than 50 years is more than obvious.

6. From EV1 to EV2 distribution

All above analyses converge to the conclusions that (a) the EV1 distribution is inappropriate for extreme rainfall and the EV2 distribution is an alternative much closer to reality, and (b) the shape parameter of the EV2 distribution can be hypothesised to have a constant value $\kappa = 0.15$, regardless of the geographical location of the raingauge station.

If the shape parameter of the EV2 distribution is fixed, its handling becomes as simple as that of the EV1 distribution. For example, the estimation of the remaining two parameters becomes similar to that of the EV1 distribution. That is, the scale parameter can be estimated by the method of moments from

$$\lambda = c_1\, \sigma \tag{3}$$

where $c_1 = \kappa / \sqrt{\Gamma(1 - 2\,\kappa) - \Gamma^2(1 - \kappa)}$ or $c_1 = 0.61$ for $\kappa = 0.15$, while in the EV1 case $c_1 = 0.78$. The relevant estimate for the method of L-moments is

$$\lambda = c_2\, \lambda_2 \tag{4}$$

where $c_2 = \kappa\, /\, [\Gamma(1 - \kappa)(2^\kappa - 1)]$ or $c_2 = 1.23$ for $\kappa = 0.15$, while in the EV1 case $c_2 = 1.443$. The estimate of the location parameter for both the method of moments and L-moments is

$$\psi = \mu/\lambda - c_3 \tag{5}$$

where $c_3 = [\Gamma(1 - \kappa) - 1] / \kappa$ or $c_3 = 0.75$ for $\kappa = 0.15$, while in the EV1 case $c_3 = 0.577$.

The construction of linear probability plots is also easy if κ is fixed. It suffices to replace in the horizontal axis the Gumbel reduced variate $z_H := -\ln(-\ln H)$ with the GEV reduced variate $z_H := [(-\ln H)^{-\kappa} - 1] / \kappa$. Such plots are portrayed in Figure 6 for the same distributions depicted in Figure 3 on Gumbel probability plots, but now for $\kappa = 0.15$. In addition to the empirical and EV2 distributions, upper and lower prediction limits of the former computed by Monte Carlo simulation have been plotted in this figure, which demonstrate the high uncertainty of estimates for large return periods.

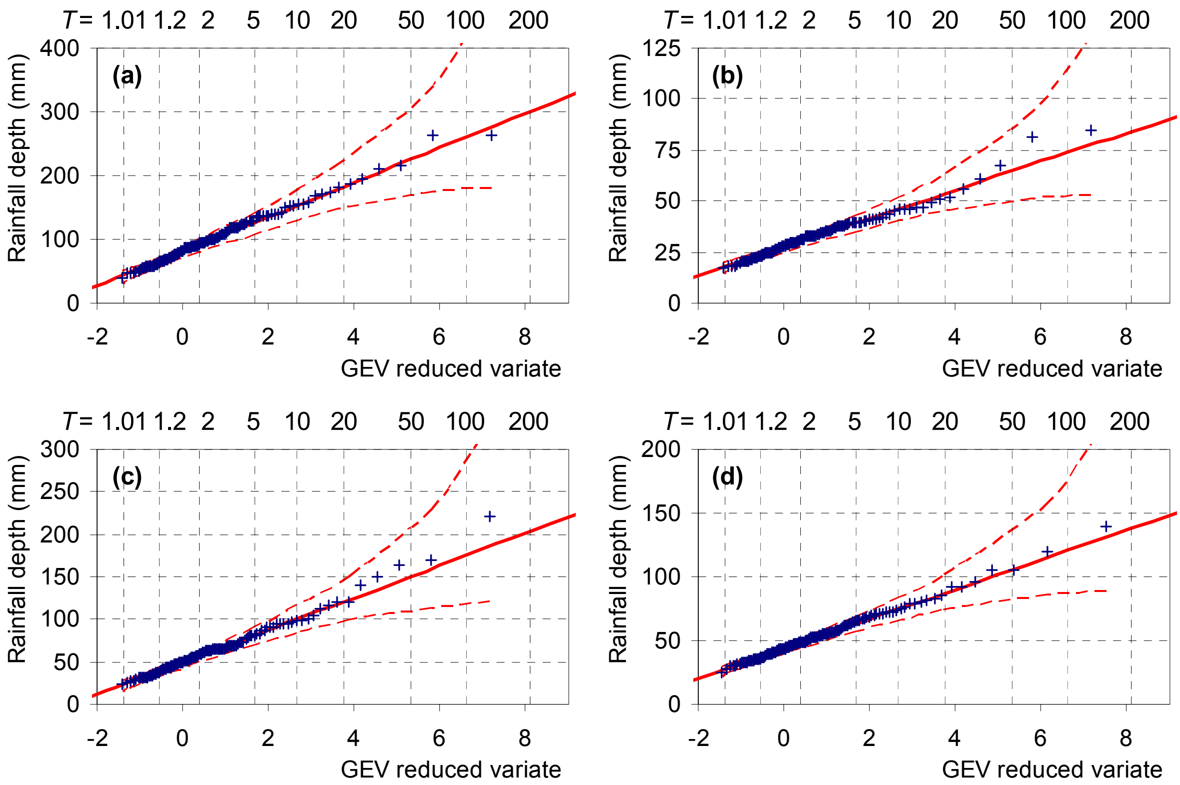

Figure 6 Empirical distributions (crosses), EV2 distributions (continuous lines), and 95% Monte Carlo prediction limits for the empirical distribution (dashed lines) of the series of (a) Charleston City, USA/SC; (b) Oxford, UK (c) Marseille, France; and (d) Florence, Italy, as in Figure 3 but in GEV plot with $\kappa = 0.15$. The EV2 distribution was fitted by the method of L-moments assuming fixed $\kappa = 0.15$.

7. Conclusions

The conclusions of this extensive analysis based on 169 rainfall series with lengths 100-154 years and a total number of 18 065 station-years from stations in Europe and USA may be summarised as follows.

1. The EV1 distribution is inappropriate for modelling extreme rainfall series, while the EV2 distribution is a choice much closer to reality.
2. The shape parameter κ of the EV2 distribution is very hard to estimate on the basis of an individual series, even in series with length 100 years or more. This is because of the estimation bias and the large sampling variability of the estimators of κ. The most important conclusion, however is that the observed variability in the values of κ in the 169 series is almost entirely explained by statistical reasons as it is almost identical with the sampling variability. This allows the hypothesis that the shape parameter of the EV2 distribution is constant for all examined geographical zones, with value $\kappa = 0.15$.
3. The location parameter ψ of the EV2 distribution turned out to be fairly constant with a mean value $\psi = 3.54$ (corresponding to $\kappa = 0.15$) and coefficient of variation as low as 0.13. However, this

small variation cannot be attributed to statistical reasons entirely as the sampling variation seems to be slightly lower than that observed in the 169 historical samples. This, however, is not a major problem as ψ can be estimated with relative accuracy on the basis of an individual series.
4. The scale parameter λ of the EV2 distribution varies with the station location and there is no need to seek a generalised law about it as it can be estimated with relative accuracy on the basis of an individual series.
5. In engineering practice, the handling of the EV2 distribution can be as easy as that of the EV1 distribution if the shape parameter of the former is fixed to the value $\kappa = 0.15$. The parameter estimation is virtually the same and very similar linear probability plots can be constructed.

The results of this study converge with other recent studies such as those by Koutsoyiannis (1999), Chaouche (2001) and Chaouche et al. (2002). Koutsoyiannis (1999) revisited Hershfield's (1961) data set (95 000 station-years from 2645 stations), and showed that this can be described by the EV2 distribution. Chaouche (2001) exploited a data base of 200 rainfall series of various time steps (month, day, hour, minute) series from the five continents, each including more than 100 years of data. Using multifractal analyses he showed that (a) an EV2/Pareto type law describes the rainfall amounts for large return periods; (b) the exponent of this law is scale invariant over scales greater than an hour; and (c) this exponent is almost space invariant.

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References

Chaouche K., 2001, *Approche Multifractale de la Modelisation Stochastiqueen Hydrologie*, thèse, Ecole Nationale du Génie Rural, des Eaux et des Forêts, Centre de Paris (http://www.engref.fr/thesechaouche.htm).

Chaouche, K., Hubert, P., Lang, G., 2002, Graphical characterisation of probability distribution tails, *Stochastic Environmental Research and Risk Assessment*, 16(5), 342-357.

Hershfield, D. M., 1961, Estimating the probable maximum precipitation, *Proc. ASCE, J. Hydraul. Div.*, 87(HY5), 99-106.

Hosking, J. R. M., 1990, L-moments: Analysis and estimation of distributions using linear combinations of order statistics, *J. R. Stat. Soc.*, Ser. B, 52, 105-124.

Jenkinson, A. F., 1955, The frequency distribution of the annual maximum (or minimum) value of meteorological elements, *Q. J. Royal Meteorol. Soc.*, 81, 158-171,.

Koutsoyiannis, D., 1999, A probabilistic view of Hershfield's method for estimating probable maximum precipitation, *Water Resources Research*, 35(4), 1313-1322.

Koutsoyiannis, D., 2003, On the appropriateness of the Gumbel distribution for modelling extreme rainfall, *Proceedings of the ESF LESC Exploratory Workshop*, European Science Foundation, Bologna, 2003.

Koutsoyiannis, D., and Baloutsos, G., 2000, Analysis of a long record of annual maximum rainfall in Athens, Greece, and design rainfall inferences, *Natural Hazards,* 22(1), 31-51.

National Research Council, 1994, *Estimating Bounds on Extreme Precipitation Events*, National Academy Press, Washington.