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Hurst-Kolmogorov dynamics in long climatic proxy records

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1. Abstract

Orbital climate theory states that the variations in insolation caused by changes in the shape of the earth's orbit (eccentricity of ellipse), tilt of the earth's axis (obliquity) and precession of the equinoxes are linked with large-scale climate variations. However, there is an on-going debate about the qualitative characteristics that describe the driving force of large scale climate dynamics (linear vs. nonlinear, insolation vs. obliquity forcing), that extends to a greater disagreement about the overall appropriateness of deterministic or stochastic descriptions of glacial cycles. Through this scientific discussion some concepts are widely used by all sides, including threshold mechanisms, state transition and multi-scale fluctuations, which are characteristics that can be associated with a power-law stochastic dependence. Hurst-Kolmogorov (HK) dynamics is a characteristic model that results in power-law dependence. Here we show that HK dynamics combined with components of orbital forcing is consistent with several proxy climatic time series spanning periods up to 500 million years before present.

2. Motivation

- There is an on-going debate about the consistency of orbital climate theory, based on some contradictions between the results of this theory and paleoclimatic data.
- In this debate, extensive use of the well-known rules of classical statistics is typically made. The Hurst-Kolmogorov approach provides a better representation of the basic statistical properties of empirical data, such as variance over different scales and autocorrelation function, as long as they indicate long-term persistence.
- Comparison between the statistical estimators of classical statistics (CS) and Hurst-Kolmogorov statistics (HKS), has shown that in these cases the variance and, therefore, the system uncertainty is underestimated by the CS (Koutsoyiannis & Montanari, 2007). This difference becomes quite serious as the Hurst coefficient, which is the index of long-term persistence strength, approaches 1.
- Temperature reconstructions of shorter scales exhibit this kind of behaviour as demonstrated e.g. by Koutsoyiannis (2003); therefore, we investigate possible HK behaviour in longer time scales.

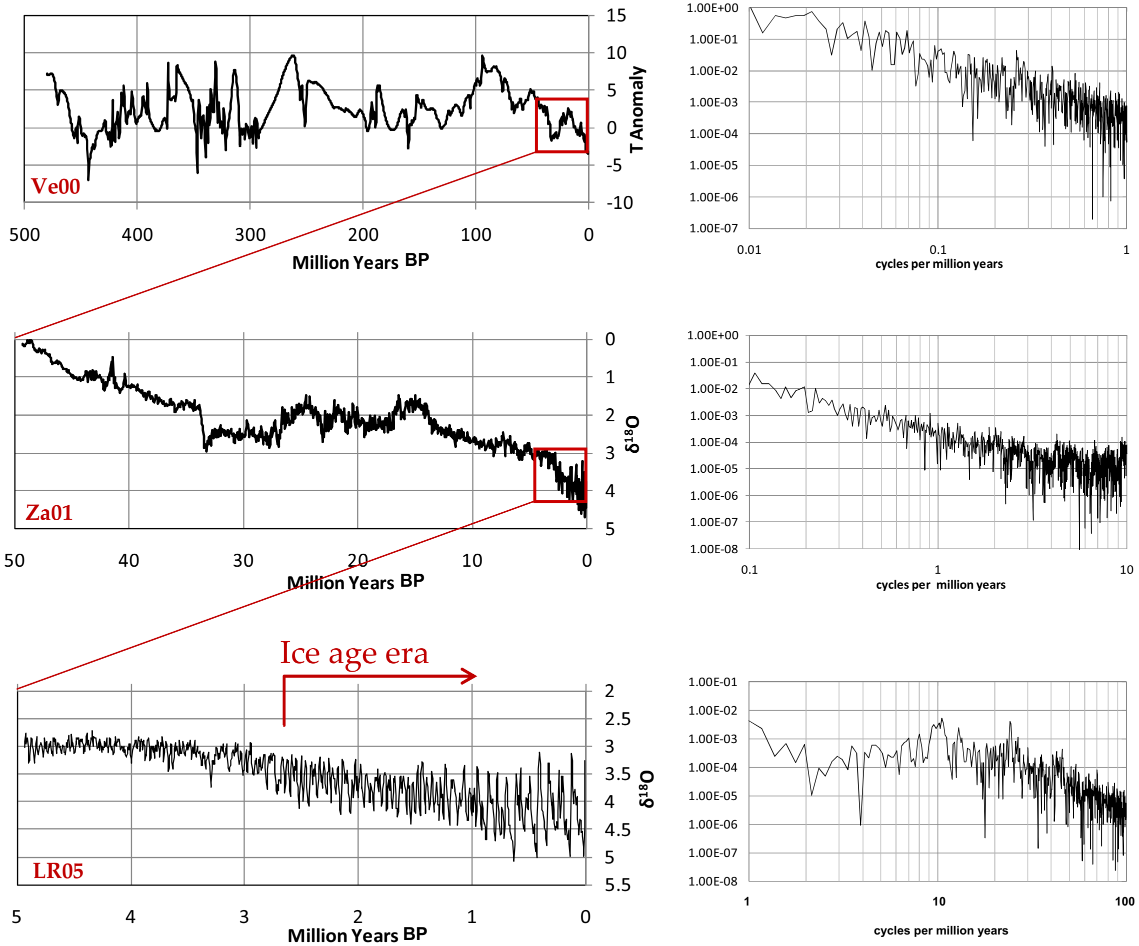
3. The Data Set

- The proxy data were of sediment origin, both of planktic and benthic type.
- All reconstructions were interpolated to lower resolution, except Hu07, because of the variable temporal resolution of the samples.
- In Hu07 though, interpolation was performed by author (Huybers, 2007).
- Hu07 age model was the only one which was not orbitally-tuned and therefore was used for validation purposes.
- Power spectrum was computed by a java FFT algorithm (www.ee.ucl.ac.uk/~mflanaga/java/)

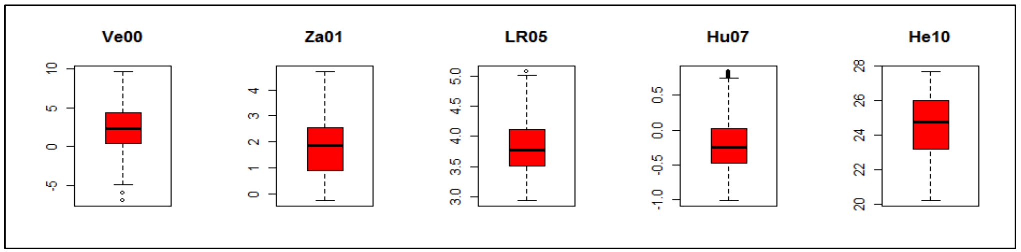
Name	Ve00	Za01	LR05	Hu07	He10
Researchers	Veizer er al.	Zachos et al.	Lisiecki & Raymo	Huybers	Herbert et al.
Year Published	2000	2001	2005	2007	2010
Parameter measured	SST	$\delta^{18}O$	$\delta^{18}O$	$\delta^{18}O$	SST
Time span (thousand years)	500 000	65 000	5 000	2 600	3 500
Resolution (thousand years)	2 – 20 000	0.2-100	1-5	1	1-4
# of records used	?	42	57	14	4
Orbital Tuned?	Yes	Yes	Yes	No	Yes
Interpolated	No	No	Yes	Yes	No
Location	Global	Global	Global	Global	Tropics

The scale factor

Power spectrum

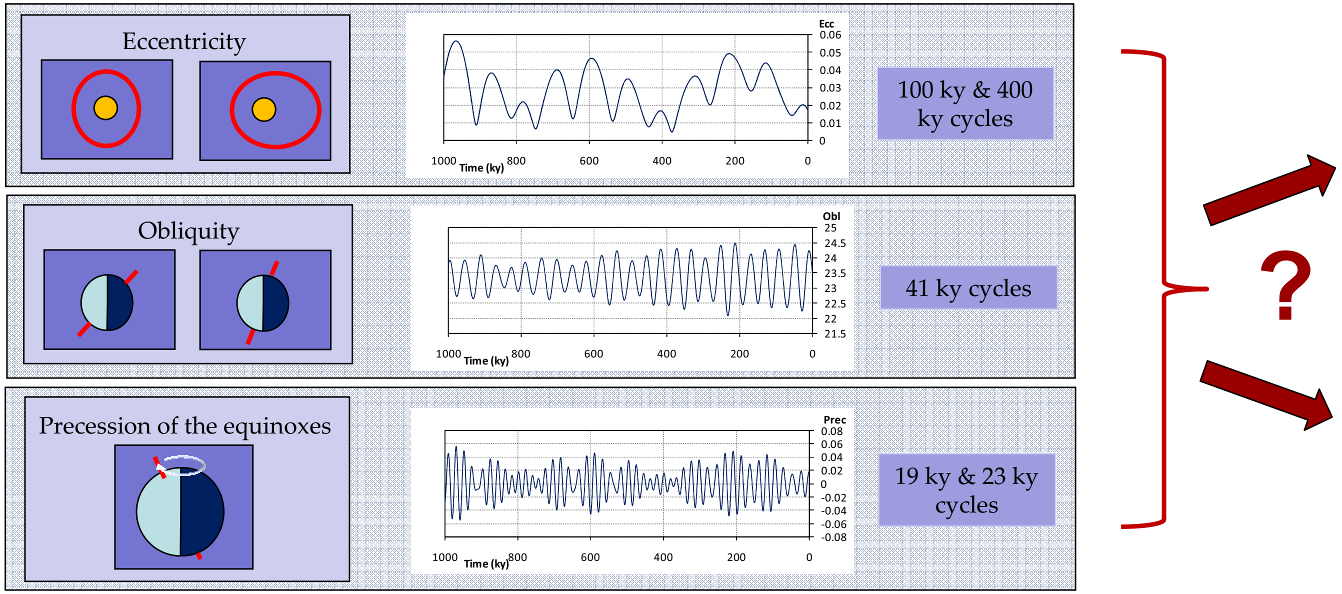


Box plots of climatic reconstructions

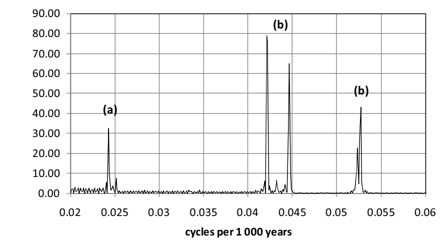


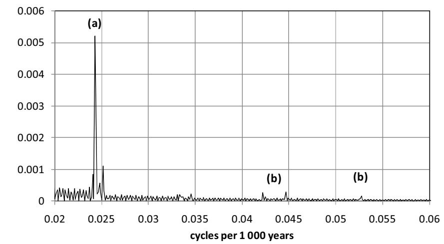
4. The orbital theory & insolation forcing

Orbital climate theory (Milankovitch cycles) is used to explain glaciations' creation and termination. The variations in earth's orbit affect the amount of insolation that our planet is receiving in each hemisphere.



Milankovitch hypothesis (1941)





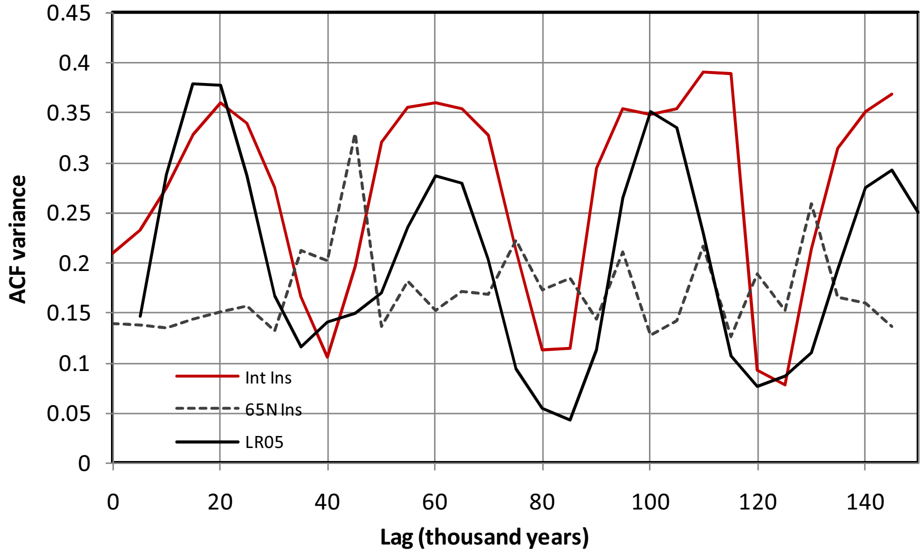
Huybers hypothesis (2006)

Milankovitch (1941) hypothesis is based on the assumption that glaciations are connected with the insolation at the summer solstice at 65ºN. Huybers (2007) proposed integrated summer insolation at 65ºN, as a more plausible explanation, due to the physical mechanics of glaciation creation and ablation.

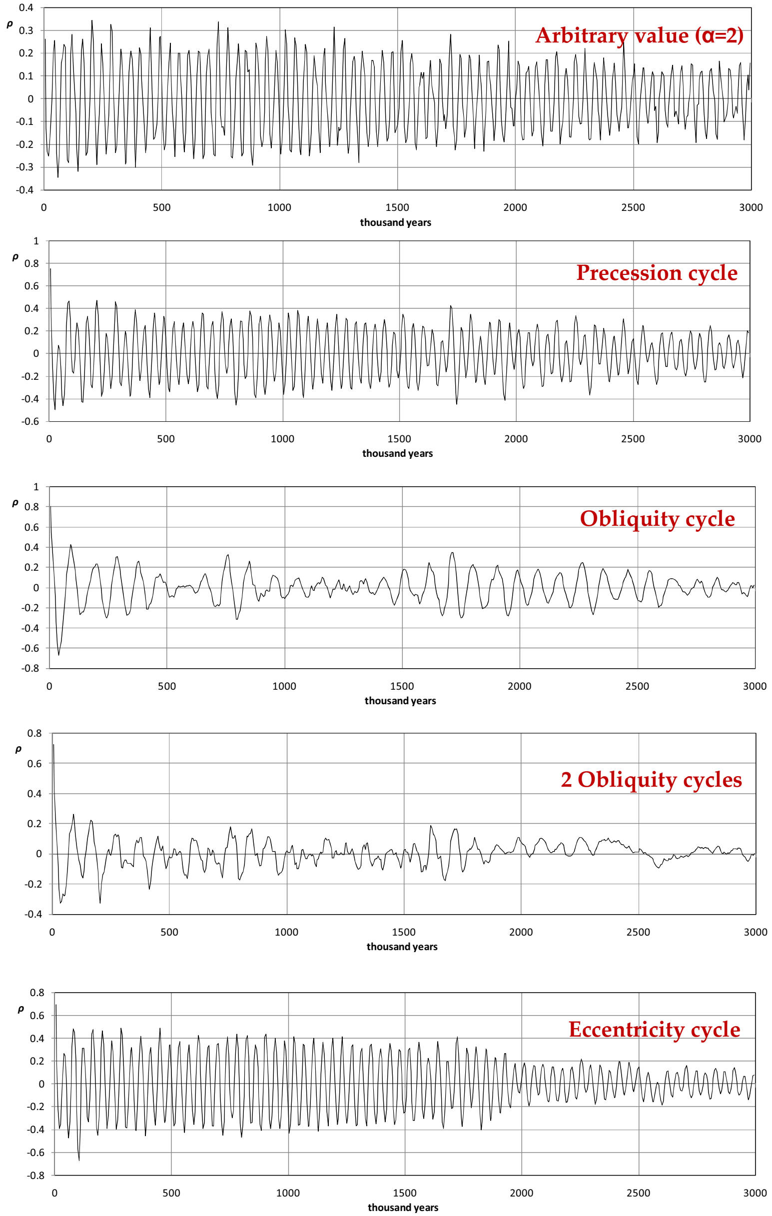
Huybers hypothesis focuses on the role of obliquity forcing, whereas Milankovitch theory is precession-oriented.

5. Autocorrelation

- We introduce a random variable $\underline{y}_i = \underline{x}_i - \underline{x}_{i-\alpha}$, where a is the time lag.
- The variance of autocorrelation function (ACF) of a periodic process with frequency $1/\kappa$ will be minimized for $\alpha = \kappa$, whereas the periodicity will remain for other values of α.
- The ACF of all paleoclimatic reconstructions of the ice age era, is minimized near 2 obliquity cycles, while it is also low just for one obliquity cycle.
- An explanation for this could be found in the fact that the 85 000-year cycle is affected also by the precession cycle, in quite a slighter way though.
- Integrated summer insolation (Huybers hypothesis), follows a similar pattern, while daily insolation does not (Milankovitch theory).



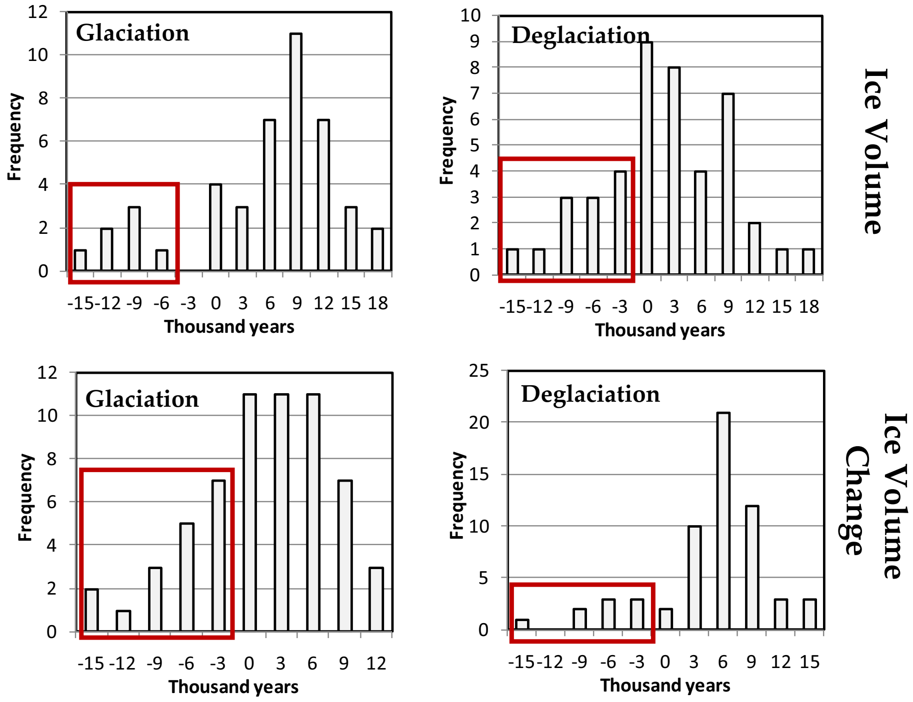
ACF of $\underline{y}_i$ for different values of α



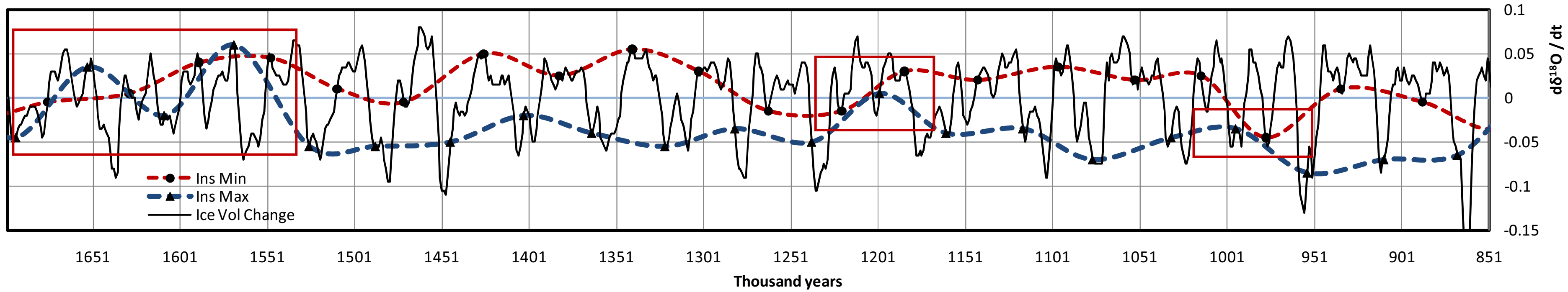
6. The limits of the deterministic approach

- Although a certain amount of glaciation happens on the rising phase of insolation and deglaciation on the opposite phase (including a 3-thousand-year uncertainty interval), there is still an important portion that is excluded from this behaviour (red rectangles), both for ice volume and ice volume change.
- This holds true only for aggregated reconstructions (Hu07 and LR05). Single proxy reconstructions show quite less correlation between insolation and glaciation phase.
- Even in aggregated reconstructions, there are certain periods that exhibit strong phase reversals.

Distance between insolation onset and (de)glaciation peak



Phase reversals in insolation/glaciation timing for Hu07 time series



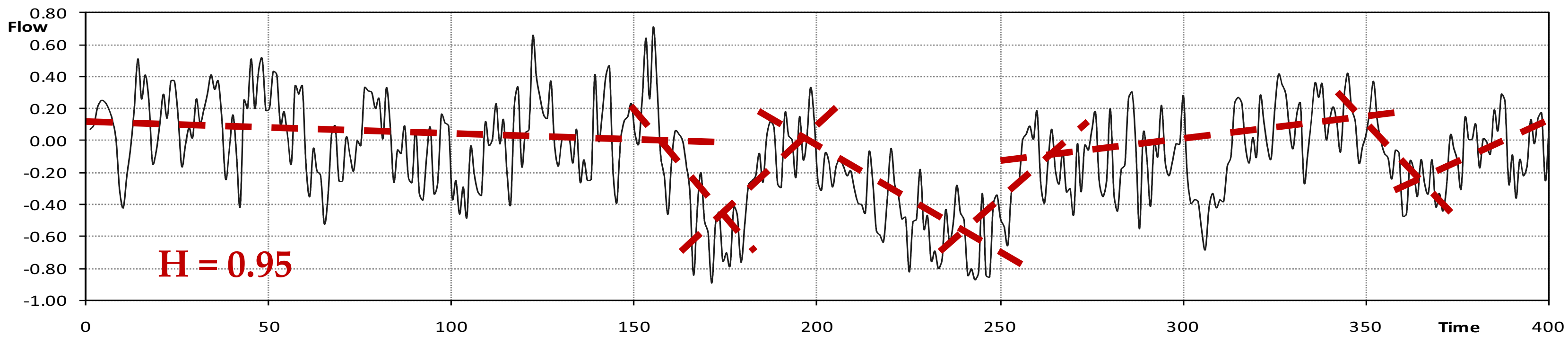
7. HK Dynamics

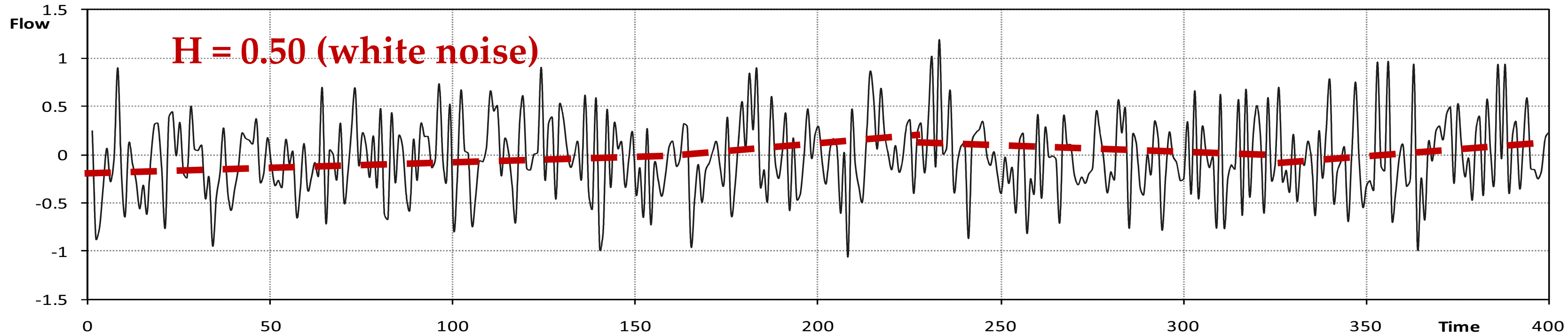
Hurst-Kolmogorov dynamics describes the scaling behaviour in natural processes. It can be perceived as the tendency of high or low values of natural events to group. Scaling behaviour can produce frequent and strong "trends" in a process, in contrast to white noise.

This behaviour is mathematically described in terms of invariance properties of the time series aggregated on different time scales, and therefore quantified through the so-called Hurst exponent, *H*, which is described by the relationship:

$$\sigma^{(k)} = k^{H-1}\sigma$$

where $\sigma^{(k)}$ and σ are the standard deviations at time scales *k* and 1, respectively. In a white noise series *H* is 0.5, whereas in real-world time series *H* is usually greater.





8. HK dynamics emergence in 3 different geological time scales

In our analysis we extend the classical relationship which describes the Hurst exponent to:

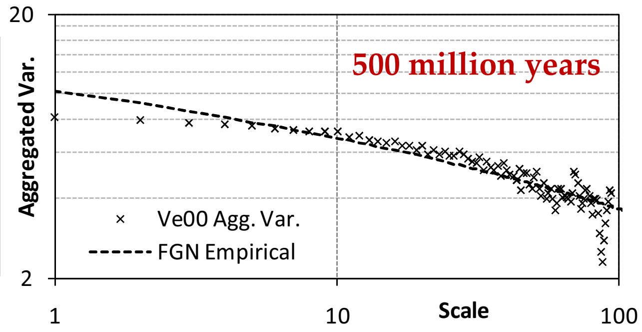
$$\sigma^{(\Delta)} = \alpha^{0.5} (\Delta + c)^{H-1} \sigma^{(1)}$$

where $\sigma^{(\Delta)}$ and $\sigma^{(1)}$ are the standard deviations at time scales Δ and 1, respectively and α, c scale coefficients. By adapting this equation to the unbiased estimator for variance proposed by Koutsoyiannis (2011), we obtain:

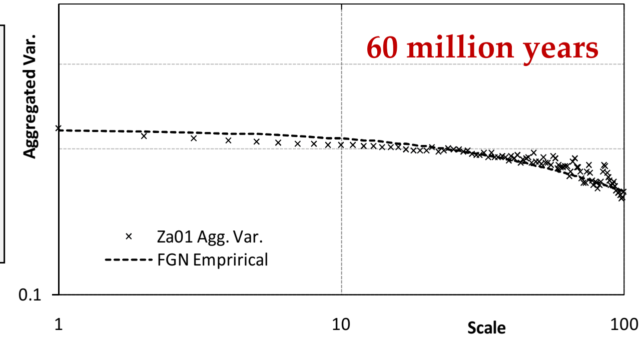
$$E[G(\Delta)] = a \frac{\Delta^2 T}{T - \Delta} \Gamma_0^{(1)} \{(c+\Delta)^{2H-2} - (c+T)^{2H-2}\}$$

where $G(\Delta)$ is the classical variance estimator (biased) at time scale Δ, $\Gamma_0^{(1)}$ is true variance at time scale 1 and T is the total observation period.

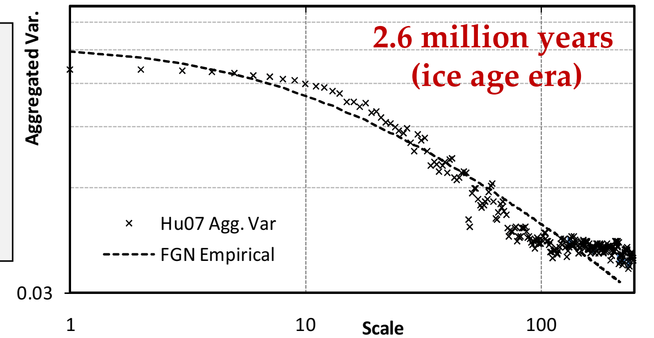
Ve00
0 – 500 mil. years
s. size = 970
H = 0.97
α = 122.94
c = 0



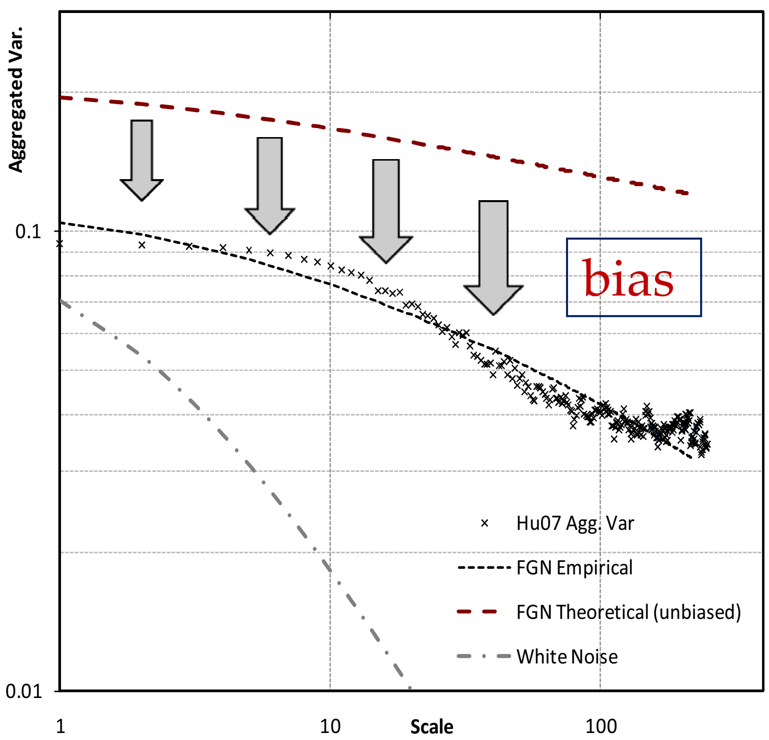
Za01
0 – 60 mil. years
s. size = 1 200
H = 0.997
α = 11.64
c = 64.13



Hu07
0 – 2.6 mil. years
s. size = 2 580
H = 0.80
α = 0.41
c = 8.78



9. Removing the harmonics



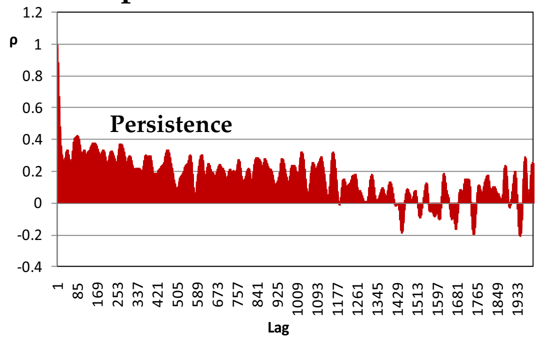
Following "Huybers hypothesis", we remove the 100 and 41 thousand years harmonics (rmse minimization-fit).

The remaining variance is 73% of the original, and the spectrum of the residual time series shows no peaks in the corresponding frequencies.

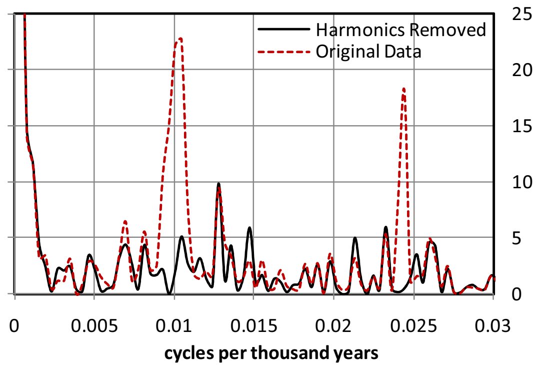
The Hurst coefficient rises from 0.80 to **0.94**, with $a = 0.22$, $c = 2.14$, which results in strong bias, averaged at approx. 300% of the empirical aggregated variance.

Even after the removal of harmonics of Hu07 and LR05, there is still distortion due to the interpolation of the data. Therefore single proxy data (used by the researchers in these reconstructions) are examined as well.

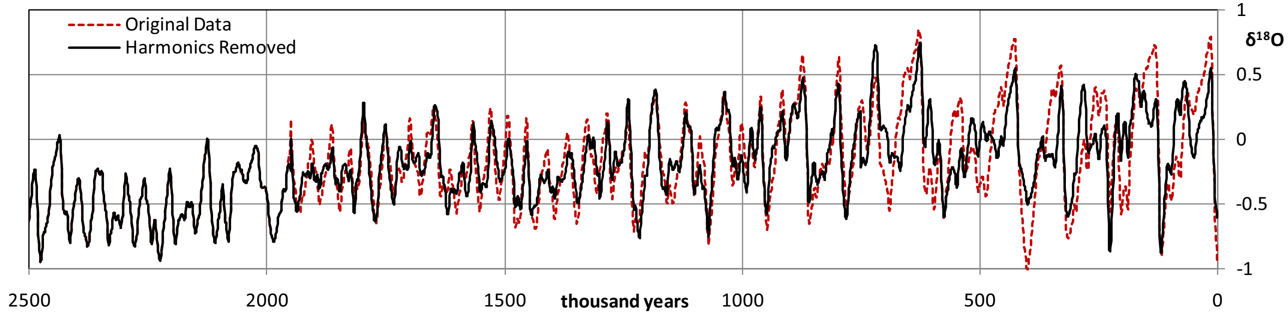
Empirical ACF



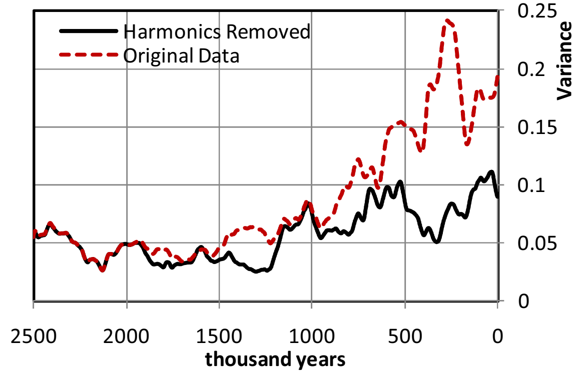
Power Spectrum



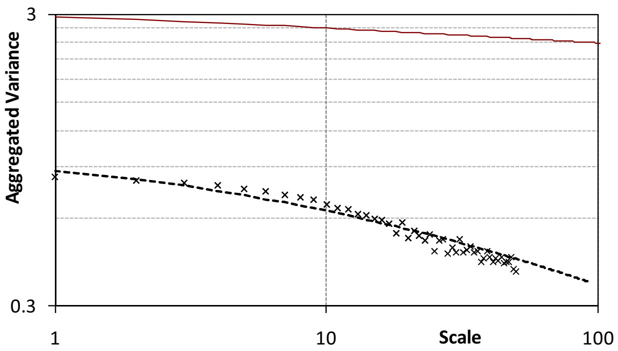
Harmonics removal transformation



Variance (200 000-year window)

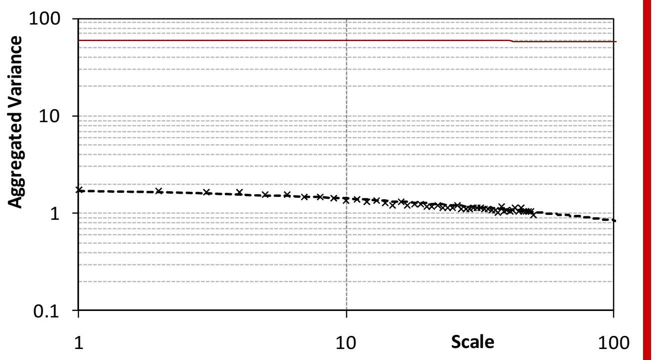


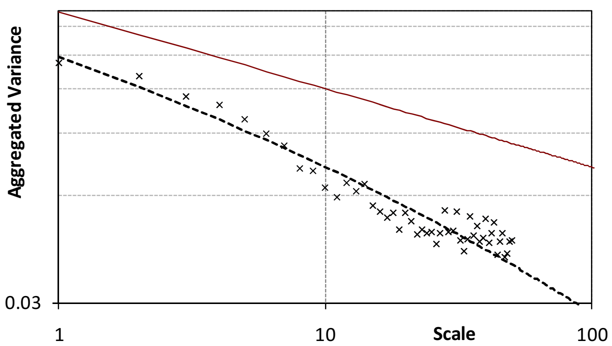
10. More evidence of HK dynamics in single proxy reconstructions



odp722
0 – 3300 th. years
s. size = 1 650
H = 0.97

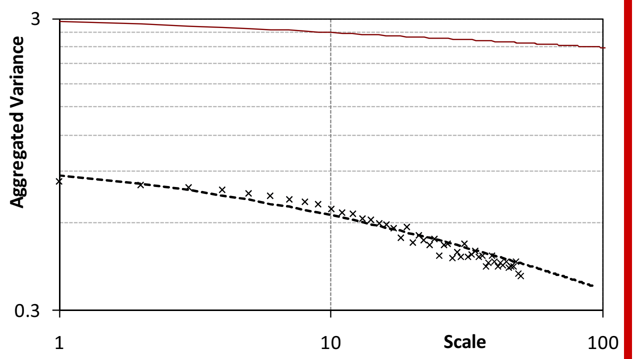
odp846
0 – 3000 th. years
s. size = 1000
H = 0.99

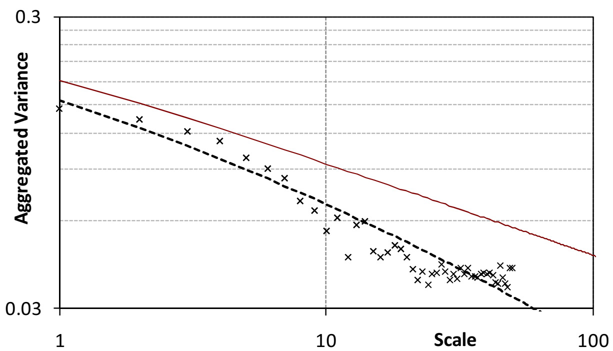




odp849
0 – 2800 th. years
s. size = 560
H = 0.89

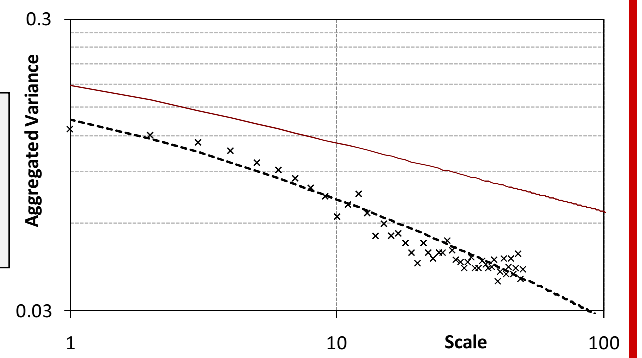
odp662
1300 – 3500 th. years
s. size = 712
H = 0.92





dsdp607
390 – 2500 th. years
s. size = 530
H = 0.84

odp677
0 – 2700 th. years
s. size = 675
H = 0.88



11. Conclusions

- We propose that self-sustained internal variability of the climate system interacts at multi-millennial scales with external forcing described by deterministic cycles of orbital origin.
- These deterministic signatures, evident in time scales between 300 to 3000 thousand years, are explained in a more satisfactory way by the "Huybers hypothesis" than the traditional "Milankovitch theory". They contribute only 30% to system variance, though, leaving 70% of variance often misinterpreted as white noise.
- The deterministic approach has some certain limits, even in the well-studied "Ice age era". Sometimes the glaciers retreat before a corresponding rise in insolation; while in other circumstances the onset of glaciation precedes the insolation fall. The same holds true for the ice volume rate of change.
- On the other hand, internal climatic variability can be described satisfactorily by HK dynamics, a stochastic process that results in power-law dependence, in scales ranging from 1 to 500 million years.
- HK dynamics has been identified in both aggregated and single proxy data, while the addition of deterministic components (orbital harmonics) has a relatively minor impact (≈ 15%) to the estimate of the Hurst coefficient.

12. References

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